

OPTIMAL VEHICLE ROUTING

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Summary: The purpose of this thesis is to design a set of minimum-cost vehicle routes, originating from material suppliers and terminating at sewing plants for the sponsor's inbound logistics network within the Southern China and ASEAN region. This thesis aims to solve a multi-vehicle capacitated static and deterministic Pickup and Delivery Problem (PDP) with time windows and multiple depots using a scientific/mathematical approach to vehicle routing so that routing decisions are optimal with respect to costs.



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KEY INSIGHTS

1. This thesis proposes a 2-index Mixed Integer Linear Program (MILP) based multi-vehicle formulation for Pick up and Delivery problems (PDP) that makes the model implementable in spreadsheet environments like MS Excel. Extant literature is lacking in exact 2-index MILP based multi-vehicle formulations for PDP thus the model proposed in this work helps bridge this gap in the literature.
2. The decision to completely eliminate or bypass a consolidation/deconsolidation hub can result in a more optimal solution for cross-boarder sourcing on certain lanes. Sponsor should explore bypassing the Bangkok hub during cross-border sourcing for the China-Thailand lane, since there is only one plant in Thailand and no deconsolidation is required. Sponsor can liaise with appropriate authority to complete customs check at checkpoints close to the Laos -Thailand border instead of the Bangkok hub. This will eliminate an entire level of network thereby saving time and costs.

Overview of the Sponsor

The Sponsor, is a U.S. based manufacturer and marketer of everyday basic apparel. It owns some of the world's strongest apparel brands and has a wide range of products that are sold in multiple countries around the world in the Americas, Asia, and Europe. The sponsor primarily operates its own manufacturing facilities either through direct ownership or fully dedicated contractors. The majority of the apparel units manufactured by the company worldwide are manufactured in such facilities.

In 2006, the company was spun off as an independent company from its parent. At that time, the company put in place a low-cost global supply chain strategy that involved restructuring the business via consolidation. The cornerstone of that strategy was:

“...moving production and operations to lower-cost countries, operating fewer and bigger facilities and aligning production flow for maximum flexibility...”

The General Problem Statement

Based on the characteristics discussed in the thesis, the problem is stated thus –

- Material is to be shipped in the amounts a_1, a_2, \dots, a_m from m shipping origins, and received in amounts b_1, b_2, \dots, b_n at each of n shipping destinations.
- Material can be routed directly to the n shipping destinations, or can first be consolidated at k hubs and de-consolidated at l hubs before reaching the n shipping destinations
- Shipping origins can be grouped into v clusters to allow milk run pick-ups.
- Clusters are dynamic and depend on the available demand as well as on the distance between shipping origins
- Materials need to reach shipping destination in p days for international shipments and q days for domestic shipments. Shipments are furthermore constrained by Saturdays, Sundays and official holidays

- The cost and distance of shipping a unit from the i^{th} origin to the j^{th} destination is known for all combinations of origins and destinations
- Other cost parameters like handling costs at each node (i.e. for loading and unloading) and the cost of using a vehicle (measured on a per vehicle basis) can be assumed to make the formulation more practical
- The fleet consists of vehicles with varying volume (cbm or cube) and weight (kg) capacity
- The objective is to reduce the overall transportation cost

From here on there are two ways to think about the problem – a holistic approach depicted in Figure 1(a) which considers the entire network as a whole, and a reductive approach as seen in Figure 1(b) which decomposes the problem (and therefore the network) into smaller parts

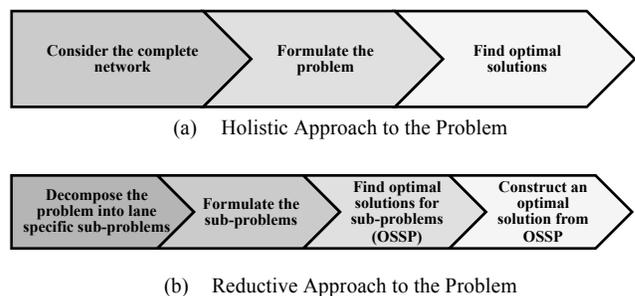


Figure 1: Thinking About the Problem

Problem Decomposition

The sponsor operates five lanes for inbound transportation. The authors considered the general problem in the context of these lanes. Lanes have been outsourced to different 3PL providers who operate independent of each other. Due to this, the problem has an optimal substructure from a practical standpoint. Therefore, a solution constructed from optimal solutions to sub-problems that address each lane will be optimal within the current framework of business rules. For the purposes of this thesis, the authors have conformed to the current business rules put in place by the sponsor for the inbound logistics operations. The solution has been proposed within that framework.

Lane-wise decomposition (see Figure 1.1, only major suppliers enumerated) simplifies the

problem by reducing the size of the network in the scope of each sub-problem (compared to the size of the entire network). The size of the problem is the main driver for selection of a solution methodology. Globally optimal solutions can be availed for the right sized problem. Herein lays the incentive for lane-wise decomposition.

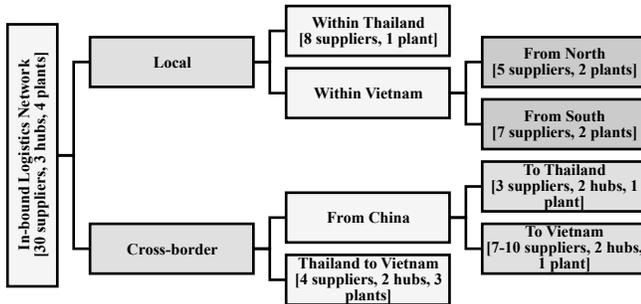


Figure 1.1: Decomposition of the Problem

1.1 The Model

- The Thai network (8 suppliers, 1 transportation hub and 1 plant) is modeled as a Graph G . Suppliers, hub and plant are nodes in the graph.
- Now generally, in order to formulate the problem as a MILP, binary decision variables of the following kind are used: x_{ij}^k . This variable is 1 if the vehicle k goes directly from node i and j in the network. It is zero otherwise.

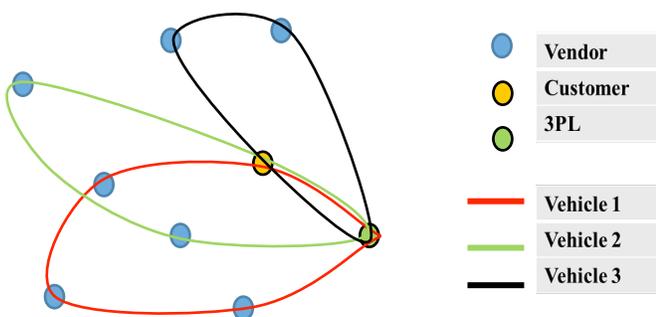


Figure 1.2: A Visual Representation of a 3-index formulation

- The problem with using such decision variables is that a three index binary variable such as this can't be implemented in a

spreadsheet environment directly.

Therefore, we have tweaked the formulation a little bit.

- We re-interpreted the trip made by each truck as one truck going back to the suppliers making one long trip.
- The problem is that the tours are no longer Hamiltonian as the truck visits 3PL and Customer again and again (lessens the constraints).
- To make the tours Hamiltonian, we introduce dummy transportation hubs. So if there are m vehicles, we introduce m dummy hubs (in addition to the 1 real hub). The idea is to tether each vehicle to a hub (dummy or otherwise) and use that hub to refer to the vehicle. So, for example: If vehicle no. 1 starts at hub 1, vehicle 2 starts at hub 2 and so on, the hub can as well represent the vehicle. So a third index is not required. In reality these are all the same hubs and this is represented in distances, service times et cetera.
- However, the above tweak modifies the tour definition. Now, m vehicles with m different tours can be modeled as a single tour by a single vehicle. Each of the m tours is represented thus –

Hub 1 → Supplier(s) → Plant → Hub 2 (tour 1 complete) → Supplier(s) → Plant → Hub 3 (tour 2 complete) → ... Plant → Hub $m+1$ (tour m complete).

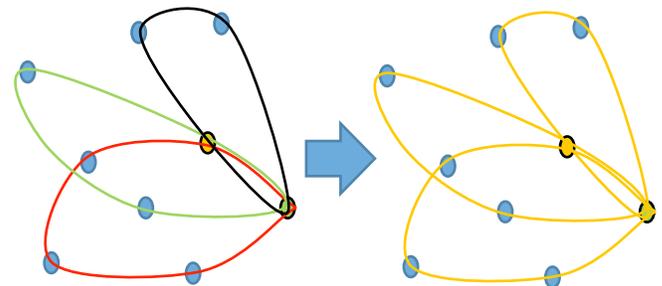


Figure 1.3: Tours by Multiple Vehicles versus Single Long Tour by One Vehicle

- Since no split loading is involved, the solution to a Pick-up and Delivery problem

is going to be such that each supplier node in the network is visited only once. We can extend this and model the entire problem such that any node (supplier, hub or plant) is visited only once. Mathematically, such a tour is called a Hamiltonian tour.

Literature Review

This thesis reviews the literature on Vehicle Routing Problems and a subclass thereof called the Pickup and Delivery Problem. The focus is on Pickup and Delivery Problems because of the nature of the central problem of this thesis. A basic classification of the Pickup and Delivery Problem, based on the existing literature, is provided. Broad categories of solution approaches for the Vehicle Routing Problem have also been discussed briefly.

Vehicle Routing is an important part of operations management and decision-making in transportation, distribution and logistics. As discussed in Chapter 2, handled correctly, it is one of the areas where companies can realize savings. Consequently, it is one of the most widely researched areas in the field of Operations Research. From a supply chain perspective, it deals with distribution of goods between suppliers/depots and customers mediated by hubs/depots while meeting certain service requirements.

The distribution of goods implies servicing the demands of –

- a **set of customers** by delivering to them
- **goods** from
- a **set of suppliers (or depots)** using
- a **set of vehicles (the fleet)**¹ operated by
- a **set of drivers** performing fleet movements over

¹ Fleet is based at one or more depots (or hubs). Some of the depots may not house the fleet but instead be used for consolidation and deconsolidation of orders picked up by the fleet.

- an appropriate subset of the **transportation network** which consists of all possible routes connecting the different network nodes or vertices (i.e. suppliers, customers and depots)

Given this, Vehicle Routing Problem (hereinafter referred to as the VRP) is a general name for a broader class of problems the solution to which must determine the aforementioned appropriate subset of routes, each undertaken by a vehicle from the fleet such that –

- customers' **demands** are fulfilled,
- certain operational **constraints** are satisfied, and;
- certain aspects (the **objective** function/s) of the transportation process are optimized

The terms presented in bold script above and on previous page are components of a typical VRP. These components are discussed further in the next section.

Methodology

In order to select the methodology for solving the problem the authors examined the largest sub-problem instance. From Figure 1.1 it can be seen that the largest definitive sub-problem instance is 'Within Thailand' with a total of 9 nodes in the network – is far smaller compared to the size of the largest unsolved instance [1].

Based on the above considerations the author decided to seek an exact solution. Using a Mixed Integer Linear Programming (MILP) based formulation was an obvious choice because as far as exact algorithmic approaches go MILP have proved to be the only workable methodology [2]. Of the three broad types of formulations – vehicle flow, commodity flow and set partitioning, a vehicle flow approach was selected because –

- commodity flow formulations have an exponential number of variables associated with each feasible circuit [3]
- set partitioning approaches also require the modelers to deal with very large number of variables [3]

Less number of variables translates into shorter

processing time. Hence the choice of using a vehicle flow formulation. In order to develop the MILP model for local sourcing the authors considered ‘Within Thailand’ lane as the reference. In section 4.10 of the thesis the motif like properties of the model built using ‘Within Thailand’ are discussed which allow for solving the other sub-problems as well.

Model Capabilities

The model developed in this thesis provides exact solution to right-sized (see section 5.1.2) multi-vehicle, capacitated, static and deterministic PDP with time windows. Its implementation in spreadsheet (e.g. MS Excel) makes it easy to use. The model incorporates order optimization by grouping orders together across suppliers to create multi-pick, single drop shipments. Therefore LTL loads (which are on the rise as per Table 2.2) from multiple suppliers can be consolidated into a single truckload to reduce costs and dock congestion at all network nodes.

The model can also be used of following purposes in its present form –

- **Heterogeneous Fleet Mix** can be handled as the model incorporates constraints that optimize routing for a fleet of vehicle with different capacities.
- **Multiple Commodities** can be modeled. Since the formulation models the many-to-many PDP as a one-to-one PDP using dummy variables, multiple commodities can be modeled by pegging each commodity to a different supplier-plant pair.
- **Multi-objective optimization** is possible by formulating the objective function as discussed in section 4.9. This allows for joint optimization of transportation costs and fleet size.
- **Extension to Green VRP** is also possible if the data on emissions and fuel efficiency is known for different vehicles.
- **Hard and soft time windows** can both be modeled depending on the user’s requirements.

- The model allows for **fleet dispatch from multiple depots**.

Conclusions

The authors set out with three objectives for this thesis.

- (a) Theoretical: Propose a mathematical formulation for the specific case of Vehicle Routing Problem encountered by the Sponsor.
- (b) Operational: Build an easy to use tool that implements the formulation to optimize the vehicle routing.
- (c) Strategic: Propose how scientific routing can result in cost reduction for the Sponsor beyond transportation.

Subsequently, a 2-index MILP based formulation has been proposed to address the specific case of VRP encountered by the sponsor thereby fulfilling the theoretical purpose of the thesis. The proposed formulation has been implemented in MS Excel spreadsheet environment in an easy-to-use format thereby fulfilling the operational objectives of the current work. The strategic purpose of this work is fulfilled by the broad-based recommendations provided in this chapter along with ideas for future research. In addition to fulfilling these objectives, the thesis document serves as an easy to understand primer to VRP for the interested beginner.

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